NPS71-88-002

NAVAL POSTGRADUATE SCHOOL

Monterey, California

A PROGRAM TO COMPUTE ELECTRIC ANOMALY
DETECTION PROBABILITIES

R. N. FORREST

OCTOBER 1988

Approved for public release; distribution is unlimited.

Prepared for:
Naval Postgraduate School
Monterey, CA 93943-5000

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral R. C. Austin
Superintendent

Harrison Shull
Provost

This report was prepared by:

Unclassified

Security Classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority 2b Declassification/Downgrading Schedule	3 Distribution Availability of Report Approved for public release; distribution is unlimited.
4 Performing Organization Report Number(s) NPS71-88-002	5 Monitoring Organization Report Number(s)
6a Name of Performing Organization Naval Postgraduate School 6b Office Symbol *(If Applicable)* 71	7a Name of Monitoring Organization Naval Postgraduate School
6c Address *(city, state, and ZIP code)* Monterey, CA 93943-5000	7b Address *(city, state, and ZIP code)* Monterey, CA 93943-5000
8a Name of Funding/Sponsoring Organization 8b Office Symbol *(If Applicable)*	9 Procurement Instrument Identification Number DIRECT FUNDING
8c Address *(city, state, and ZIP code)*	10 Source of Funding Numbers Program Element Number / Project No / Task No / Work Unit Accession No

11 Title *(Include Security Classification)*

A PROGRAM TO COMPUTE ELECTRIC ANOMALY DETECTION PROBABILITIES

12 Personal Author(s) R. N. Forrest

13a Type of Report	13b Time Covered	14 Date of Report *(year, month,day)*	15 Page Count
Technical	From To	October 1988	64

16 Supplementary Notation

17 Cosati Codes			18 Subject Terms *(continue on reverse if necessary and identify by block number)*
Field	Group	Subgroup	Electric Anomaly Detection, Lateral Range Function, Detection Models, Electric Dipole

19 Abstract *(continue on reverse if necessary and identify by block number)*

The report contains user instructions, a listing and documentation for a microcomputer BASIC program that can be used to compute an estimate of the probability that an electrometer based electric anomaly detection (EAD) system will detect a submarine during an encounter.

20 Distribution/Availability of Abstract [X] unclassified/unlimited [ ] same as report [ ] DTIC users	21 Abstract Security Classification Unclassified
22a Name of Responsible Individual R. N. Forrest	22b Telephone *(Include Area code)* (408) 646-2653 / 22c Office SymbolT. 55Fo

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted security classification of this page

All other editions are obsolete Unclassified

The program in this report is presented without representation or warranty of any kind.

Table of Contents

I. Introduction

This report contains user instructions, a listing and documentation for a microcomputer BASIC program that can be used to compute an estimate of the probability that an electrometer based detection system will detect a submarine electric dipole field during an encounter.

The program generates detection probabilities based on two encounter models. In the first encounter model, the detection system uses a crosscorrelation detector. In the second encounter model, the detection system uses a square law detector. Relative to operationally realizable values, probabilities based on the first model represent upper bounds and those based on the second model represent lower bounds. For both encounter models, the signal is proportional to the square of the component of the electric field that is measured by a single axis electrometer in an infinite medium of constant conductivity and the electric field noise does not change with changes in the position of the electrometer. Also, for both encounter models, the encounter is a straight line encounter with constant vertical separation.

The encounter models can be interpreted as models of a electric anomaly detection system that is moving with constant course, speed and altitude in an encounter with a submarine moving with constant course, speed and depth. Or, they can be interpreted as models of a stationary electric anomaly detection system in an encounter with a submarine moving with constant course, speed and depth.

The program parameters include: electrometer axis direction, electrometer axis depression angle, encounter medium conductivity, submarine electric moments, submarine course, speed and altitude, electrometer course, speed and depth, encounter lateral range (the horizontal range at the closest point of approach in a straight line encounter) and false alarm rate. (In the program, a false alarm is the event that the detection system classifies noise as a dipole signal.)

II. Program User Instructions

As listed in Appendix 10, the program can be run under a BASIC language that is compatible with IBM PC BASIC. If the listing is used to enter the program through the keyboard, then the program should be saved with the name EAD.BAS or the value of N$ on line 40 should be changed to the file name under which it is saved.

The program contains user instructions in the form of query and parameter limitation messages. As an example of the former, after starting the program under BASIC, the following message should appear:

Electric Anomaly Detection (EAD) Lateral Range Function

generate or print a program data file (g/p)?

By entering p, data can be printed from a program data file that was generated by the program. By entering g, a program data file can be generated for a set of user specified conditions. With either response, a sequence of additional queries is displayed. These queries require either an indicated response or a parameter value as the input. If the initial response is g, the sequence includes queries whose responses determine whether or not an auxiliary data file will be generated that can be used for future input of magnetic, processing or kinematic data. In particular, the first query in the sequence gives the option of using a combined electric, processing and kinematic data file. If the response indicates that it should be used and the file is available, the parameter values that remain to be entered in order to generate a program data file are the following: the false alarm rate, the electric field noise, the maximum encounter lateral range and the lateral range step. The combined file should be used only if the effect of varying just one or more of these parameters is desired. If the response indicates that the file will not be used, queries concerning electric, kinematic and processing parameter values are displayed.

After all of the program parameter values have been entered, a query is displayed giving the option of generating the combined file. Then a query giving the option of generating a program data file, a query giving the option of printing the encounter parameter values and a query giving the option of printing lateral range function values are displayed. The lateral range function values are the encounter detection probabilities indexed by lateral range. The parameters maximum lateral range and lateral range step determine the index lateral ranges of the encounters for which probabilities of detection are computed.

The program generates electric signal values that correspond to points in time during an encounter. Following the lateral range function query, a query is displayed that gives the option of printing the electric signal values for an encounter. If the option is exercised, the option is repeated. When the option is not exercised, a query is displayed that gives the option of generating or printing a new program data file. If this option is not exercised, the program ends.

Some suggested guides for determining parameter values can be found in Section IV of this report.

III. Encounter Model Limitations

In the two encounter models, the signal is the unfiltered signal that is generated by an electric dipole that moves relative to an electrometer. Depending on the input filtering, describing a submarine electric anomaly field as an electric dipole field should not be a significant limitation for an encounter slant range at the closest point of approach (CPA) that is greater than one hull length. The detection decision is based on samples from a single time interval (window) that is centered on the CPA. The length of the sampling interval and the sampling rate are parameter values that are inputs to the program. In terms of signal-to-noise ratio, there is an optimum sampling interval length (integration time) and sample rate. Although dipole signal energy is not symmetrically distributed about the CPA time, for a given sampling interval, the difference between the signal energy for the optimum interval location and the CPA centered location should not be significant in most cases.

The encounter model electrometer noise samples are determined by a gaussian random process and they are values of independent identically distributed normal random variables. The standard deviation of these random variables is referred to as the electric noise and the variance is the noise in the sense of the signal-to-noise ratio. The degree of correspondence between this process and operational noise depends on the nature of the dominant operational noise sources and on the electrometer input filtering (noise whitening).

In the encounter models, the intervals are adjacent but not overlapping and a detection statistic corresponds to each sample interval and its value is determined by the sample values. If the value of the detection statistic for an interval equals or exceeds a threshold value, a detection is indicated. The threshold value is determined by the false alarm probability which in turn is determined by the false alarm rate and the sampling interval length.

A detector that used a moving sample interval that was generated by replacing the oldest sample by the newest one would correspond more closely to the detector in an operational detection system. In a model of the detector, since the sample windows overlap, the detection statistics would represent a sequence of random variables that were correlated over an interval equal to the width of the sample interval. Because of this dependence, it seems unlikely that the results that would be obtained with an encounter model based on an overlapping interval detector would differ significantly from those obtained with an encounter model based on a nonoverlapping interval detector.

For encounter lengths of the order of a few nautical miles or less, the straight line encounter condition should not be a significant limitation. In particular, this should be the case for a fixed electrometer since, for a submarine target, vertical separation and course changes should be less likely to occur.

Other models are available that can be used as the basis for computing an estimate of the probability that an electric anomaly detection system will detect a submarine during an encounter. For example, one that is described in Appendix 6 could be used to determine the slant ranges of straight line encounters for which the detection probability is equal to a specified value. The parameter values that are required to do this are an average submarine electric dipole moment, a detection system capability factor and a noise factor. Values for these parameters can be determined from operational data. However, the values are specific to averages over a particular set of encounter conditions. An advantage of the two encounter models relative to this model is their adaptability to different electric, processing and kinematic conditions.

IV. Parameter Values

The electric parameter value queries are generally explanatory with regard to the value that should be entered. This is also true of the kinematic parameter value queries. However, there is some ambiguity with respect the processing parameter value queries and the noise parameter value query. To reduce this ambiguity, a brief discussion of the common characteristics of the two encounter model processing parameters and the noise parameter is given below. This is followed by some guidelines for choosing these parameter values.

In both encounter models, a decision is made at the end of each sampling interval. The decision is either noise energy was present during the interval or noise energy plus signal energy was present during the interval. The sample intervals are adjacent, equal width, nonoverlapping time intervals. The number of samples that are input in a sample interval is determined by the sampling rate and the interval length.

The program default choice for the sampling rate is $2 \cdot \mathrm{MAXF}$ where MAXF is a parameter that is labeled the maximum electric signal frequency. This sample rate is the Nyquist rate for an ideal low pass filter. However, the signal in a sample interval that is computed by the program represents an unfiltered dipole signal. This is a reasonable approximation if the signal energy that is associated with signal components greater than MAXF is relatively small. As discussed below, the noise energy in a sample interval should be considered to be proportional to MAXF in order to be consistent with the encounter models. Ideally, a default choice for MAXF would make the ratio of the signal energy to the noise energy a maximum for the sampling interval of an encounter. The program default choice for MAXF is 2·MAXVM/MINR0 where MAXVM is a user estimated maximum encounter relative speed converted from knots to meters per second and MINR0 is a user estimated maximum slant range at CPA in meters in terms of a just detectable target. (A precise definition of a just detectable target can be made in terms of a specified detection probability, false alarm probability and target dipole moment.) The default choice for MAXF is consistent with the observation in Reference 1 that if an optimum value for MAXF is determined for a minimum dipole moment target, then no significant increase in MAXF is required in order to maintain a required detection probability if the encounter lateral range is decreased even though the signal energy spectrum is shifted to higher frequencies.

Because of the detection statistics and the gaussian noise model, if there were no penalty for decision delay, a sample interval length for a signal should be chosen equal to the signal duration, since this would make the detection probability for an encounter a maximum. In the program, a default choice for the sample interval length (integration time) is 2·MAXR0/MINVM where MINVM is a user estimated maximum encounter relative speed converted from knots to meters per second and MAXR0 is a user estimated maximum slant range at CPA in meters for a detectable encounter (in terms of a specific detection probability and false alarm probability) with a user specified maximum dipole moment target. This choice might be considered a balance between minimizing decision delay and maximizing detection probability. In the encounter models, the sample intervals are located so that the CPA time is at the center of a sample interval and this sample interval is the only one that contains signal energy. This characteristic is consistent with the default choice for the sample interval. For a given sample interval length, although in general the interval location is not the optimum one in terms of signal energy, it should be approximately so in most cases.

The program noise parameter is SIG. It represents the standard deviation σ associated with the electric noise process of the two encounter models. In terms of the ideal low pass filter implied by the encounter models, its square should be equal to $\mathrm{MAXF} \cdot (\mathrm{SIG0})^2$ where SIG0 is the electric noise process constant power spectral density. The program does not enforce this relation. Therefore, in using the program, the implied relation between the two input parameters: electric noise and maximum electric signal frequency should be kept in mind. If an average value of the peak-to-peak electric noise for an encounter can be estimated, for example from a electrometer trace, then the value for SIG should be chosen so that the estimate is 4 to 6 times this value.

Appendix 1. The Detection Statistics

In this appendix, $y_1, y_2, \cdots, y_m$ are sequential values (voltages) representing the sample values in a sample interval. They are the input to an electrometer's detector. With these m sample values, the detector computes the value of a detection statistic. This value is represented by x and the detection decision corresponding to the sample interval is determined by the decision rule: If $x \geq x^*$, then the input during the sample interval was noise plus signal, otherwise, the input was noise.

For both encounter models, the detection probability and the false alarm probability are decreasing functions of x^* and the relation is one-to-one in both cases. In the program, the false alarm probability p_f is used to determine a unique value of the threshold x^*. This value is then used to determine a unique value of the detection probability p_d.

In the program, p_f is found using the relation $p_f = R \cdot \delta t$ where R is the false alarm rate in false alarms per second and δt is the sample interval length in seconds. This relation is based on the following argument: With no signal energy in a sample interval, $y_1 = n_1, y_2 = n_2, \cdots, y_m = n_m$ where $n_1, n_2, \cdots, n_m$ are noise values (voltages) input to an electrometer's detector. The n_i are values of independent normal (gaussian) random variables, each with mean zero and standard deviation σ. Because of this, in the encounter models, values of x for different sample intervals are the values of independent random variables that determine two outcomes: $x \geq x^*$ or $x < x^*$.

Therefore, in terms of these outcomes, an encounter is a sequence of independent Bernoulli trials. If there is no signal present, since the noise is determined by a stationary process, p_f is the same for each sample interval and, in terms of these outcomes, the sequence is a series of repeated independent Bernoulli trials and therefore the expected number of trials between false alarms is $1/p_f$. Since the time between trials is δt, the expected number of seconds between false alarm is $\delta t/p_f$ or the expected number of false alarms per second R is equal to $p_f/\delta t$.

The determination of x^* depends on the encounter model statistic. For both encounter models, when there is a signal, $y_1 = n_1 + s_1$, $y_2 = n_2 + y_2$, $\cdots$, $y_m = n_m + s_m$ where s_1, s_2, $\cdots$, s_m are signal values (voltages) input to a electrometer's detector. The models imply that the signal values $s_i = K \cdot (E_s)_i$ where K is a constant whose value is determined by the characteristics of the encounter electrometer and where the $(E_s)_i$ are dipole electric signal intensities. The models also imply that the noise values n_i are determined by a gaussian stochastic processes characterized by a standard deviation σ and that $n_i = K \cdot (E_N)_i + n_i'$ where the $(E_N)_i$ are electric noise intensities and the n_i' are electrometer instrument noise values (voltages). In the program, the magnitude of K is 1. Since, for both models, p_d depends only on the ratio of signal energy to noise energy for a sample interval, this is a satisfactory choice for the program if instrument noise is assumed to be determined by a process independent of the electric noise process and to be expressed in terms of an equivalent electric noise $(E)_i' = (1/K)\cdot n_i'$.

For both encounter models, the signal (the average signal power) $S = (1/m)\cdot\Sigma\, s_i^2$ and the noise (the expected value of the average noise power) $N = \sigma^2$ so that the signal-to-noise ratio is $(1/m)\cdot\Sigma\, s_i^2/\sigma^2$ where the sum index $i = 1, 2, \cdots, m$.

The Crosscorrelation Detector Statistic: The statistic for the first encounter model is a crosscorrelation detector statistic that is defined by the sum

$$x = \Sigma\, y_i \cdot s_i$$

where the summation index $i = 1, 2, \cdots, m$ and the sum is over the values corresponding to a sample interval. For the first encounter model, the characteristics of both the noise and the signal are required in order to determine encounter detection probabilities. In particular, the signal values for an encounter are in the memory of the detector prior to the encounter. For the encounter conditions and a specified false alarm probability, the statistic is optimum in the sense that the encounter detection probability for this statistic is at least equal to that for any other statistic. Because of these considerations, encounter probabilities based on the crosscorrelation statistic can be considered to represent upper bounds on detection performance against dipole targets for electrometers of the type described by the models.

For a sample interval without signal energy, x is the value of a normal random variable with a mean $\mu_X = 0$ and a variance

$\sigma_X^2 = \sigma^2 \cdot \Sigma\, s_i^2$ where σ is the standard deviation associated with the noise process and the sum index $1 = 1, 2, \cdots, m$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_f = 1 - P(x^*/\sigma_X)$$

where $P(z)$ is the standard normal cumulative distribution function. This relation is the basis for determining the threshold value x^*.

For the sample interval with signal energy, x is the value of a normal random variable with a mean $\mu_X = \Sigma\, s_i^2$ where the sum index $i = 1, 2, \cdots, m$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_d = 1 - P(v^* - d^{\frac{1}{2}})$$

where $v^* = x^*/\sigma_X$ and $d = \Sigma\, s_i^2/\sigma^2 = (1/m) \cdot S/N$. This relation is the basis for determining encounter detection probabilities for the first encounter model. The relation implies that for a specified false alarm probability p_f the detection probability p_d is an increasing function of the signal to-noise ratio S/N.

The Square Law Detector Statistic: The statistic for the second encounter model is a square law (energy) detector statistic that is defined by the sum

$$x = \Sigma\, y_i^2$$

where the sum index $i = 1, 2, \cdots, m$ and the sum is over the values corresponding to the sample interval. For the second encounter model, only the characteristics of the noise are required to determine encounter detection probabilities.

For a sample interval without signal energy, x/σ^2 is the value of a chi-square random variable with m degrees of freedom. This implies that

$$p_f = 1 - P(x^*/\sigma^2 | m)$$

where $P(x^*/\sigma^2 | m)$ is the chi-square cumulative distribution function for a chi-square random variable with m degrees of freedom and where σ is the standard deviation associated with the noise process. This relation is the basis for determining the threshold value x^*.

For the sample interval with signal energy, x/σ^2 is the value of a noncentral chi-square random variable with m degrees of freedom and noncentral parameter $\Sigma\, s_i^2/\sigma^2$ where the sum index $i = 1, 2, \cdots, m$ and the sum is over the values corresponding to the sample interval. This implies that

$$p_d = 1 - P(x^*/\sigma^2 | m, \Sigma\, s_i^2/\sigma^2)$$

where $P(x^*/\sigma^2 | m, \Sigma\, s_i^2/\sigma^2)$ is the noncentral chi-square cumulative distribution function for a noncentral chi-square random variable with m degrees of freedom and noncentral parameter $\Sigma\, s_i^2/\sigma^2 = (1/m) \cdot S/N$. This relation is the basis for determining encounter detection probabilities for the second encounter model. The relation implies that for a specified false alarm probability p_f, the detection probability p_d is an increasing function of the signal-to-noise ratio S/N. This is made more evident by the following relation:

$$P(x^*/\sigma^2 | m, \Sigma\, s_i^2/\sigma^2) = \Sigma\{(a^j/j!) \cdot \exp(-a) \cdot P[x^*/\sigma^2 | (m + 2 \cdot j)]\}$$

where the sum index $j = 0, 1, 2, \cdots$ and the parameter

$a = (1/2) \cdot \Sigma\, s_i^2/\sigma^2 = (m/2) \cdot (S/N)$ where the sum index $i = 1, 2, \cdots, m$. Note that the relation $p_d \geq p_f$ is satisfied, since $P(x^*/\sigma^2 | m) \leq P[x^*/\sigma^2 | (m + 2 \cdot j)]$ for $j = 0, 1, 2, \cdots$.

Appendix 2. Program Probability Calculations

The program evaluates the cumulative and inverse cumulative distribution functions using approximations that are described in Reference 2. These approximations are listed below.

The Standard Normal Cumulative Distribution Function

Approximation:

$$P(z) = 1 - s \cdot t \cdot (b_1 + t \cdot (b_2 + t \cdot (b_3 + t \cdot (b_4 + t \cdot b_5))))$$

where $s = (1/(2 \cdot \pi)^{\frac{1}{2}}) \cdot \exp(-z^2/2)$ and $t = 1/(1 + b_0 \cdot z)$. And where

$b_0 = .2316419$, $b_1 = .319381530$, $b_2 = -.356563782$,

$b_4 = -1.821255978$, $b_5 = 1.330274429$,

and $z \geq 0$. For $z < 0$, $P(z) = 1 - P(|z|)$.

The Inverse Standard Normal Cumulative Distribution Function

Approximation:

$$z(P) = t - (c_0 + t \cdot (c_1 + t \cdot c_2))/(1 + t \cdot (d_1 + t \cdot (d_2 + t \cdot d_3)))$$

where $t = (\ln(1/Q^2))^{\frac{1}{2}}$ and $Q = 1 - P$. And where

$c_0 = 2.515517$, $c_1 = .802853$, $c_2 = .010328$,

$d_1 = 1.432788$, $d_2 = .189269$, $d_3 = .001308$,

and $.5 \leq P < 1$. For $0 < P < .5$, $z(P) = -z(1 - P)$.

The Inverse Chi-Square Cumulative Distribution Function

Approximation:

$$v(P|m) = m \cdot [1 - 2/(9 \cdot m) + z \cdot (2/(9 \cdot m))^{\frac{1}{2}}]^3$$

where $P(z) = P(v|m)$. In the program, the inverse standard normal cumulative distribution function approximation is used to determine z.

The Noncentral Chi-Square Cumulative Distribution Function Approximation:

$$P(w|m, \Sigma\ s_i^2/\sigma^2) = P(z)$$

where $z = [2 \cdot w/(1 + b)]^{\frac{1}{2}} - [2 \cdot a/(1 + b) - 1]^{\frac{1}{2}}$ with $a = m + \Sigma\ s_i^2/\sigma^2$, $b = (\Sigma\ s_i^2/\sigma^2)/(m + \Sigma\ s_i^2/\sigma^2)$ and the sum index $i = 1, 2, \cdots, m$. In the program, the standard normal cumulative distribution function approximation is used to determine $P(z)$.

Appendix 3. The Electric Signal

The encounter models are defined by the following conditions: A submarine electric anomaly field is an electric dipole field. An electrometer electric signal value is the magnitude of the component of a dipole electric field in the direction of the electrometer axis. The basis for determining electric signal values in the program is an expression that can be developed as follows: In the right handed rectangular coordinate system that is shown in Figure 1, an electric dipole is at the origin, the xy-plane is the horizontal plane at a representative point in an encounter region, the positive y-axis is in the direction τ of the electrometer's axis and the positive z-axis is positive upward. In this rectangular coordinate system, the unit vector with the orientation of the electrometer's axis can be expressed as $\mathbf{e} = \mathbf{j}\cdot\cos\Phi + \mathbf{k}\cdot\sin\Phi$ where Φ is the electrometer axis depression angle.

In a spherical coordinate system with the origin at the electric dipole and the polar axis in the direction of the dipole moment, $\mathbf{E}_d = (c\cdot p/r^3)\cdot(2\cdot\mathbf{r}\cdot\cos\Theta + \boldsymbol{\Theta}\cdot\sin\Theta)$ is the electric field of the dipole at a point with spherical coordinates (r,Γ,Θ). In this expression p is the magnitude of the dipole moment and c is a constant whose value is determined by the choice of units. The electric signal for an electrometer that is described by the encounter models is $E_s = \mathbf{e}\bullet\mathbf{E}_d$ and it can be expressed in terms of the rectangular coordinate system as

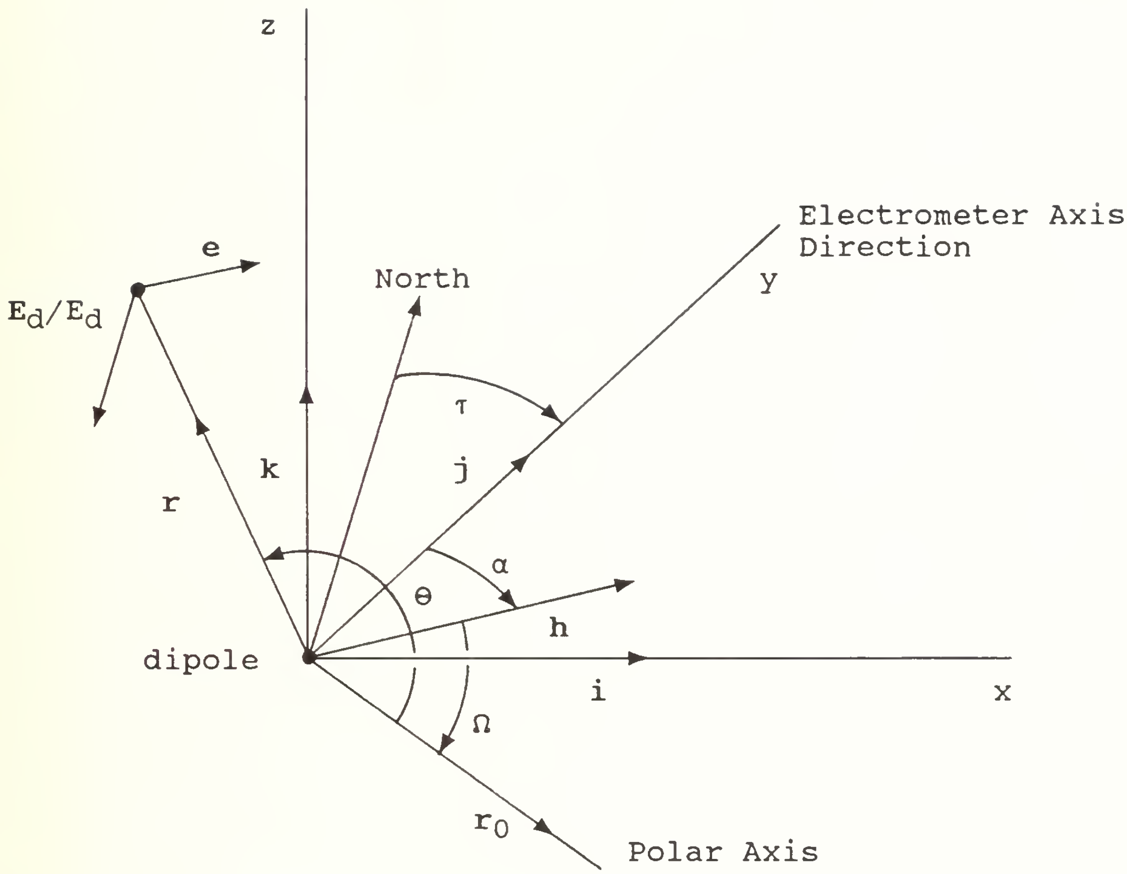

Figure 1. A unit vector in the direction of the electrometer axis and a unit vector in the direction of the dipole field of a dipole at the origin are shown at a point that is a distance r from the dipole. The unit vector **h** is in the direction of the horizontal component of the dipole moment.

follows: First, let $\mathbf{r_0}$ be a unit vector with the orientation of the spherical coordinate system polar axis. Then the electric dipole field $\mathbf{E_d} = (c \cdot p/r^3) \cdot (3 \cdot \mathbf{r} \cdot \cos\theta - \mathbf{r_0})$ since the unit vector $\mathbf{r} = (\mathbf{i} \cdot x + \mathbf{j} \cdot y + \mathbf{k} \cdot z)/r$ where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, $\boldsymbol{\theta} = [(\mathbf{r_0} \times \mathbf{r}) \times \mathbf{r}]/\sin\theta = [(\mathbf{r} \bullet \mathbf{r_0}) \cdot \mathbf{r} - (\mathbf{r} \bullet \mathbf{r}) \cdot \mathbf{r_0}]/\sin\theta$, and the dot product $\mathbf{r} \bullet \mathbf{r_0} = \cos\theta$. The unit vector $\mathbf{r_0}$ can be expressed in the rectangular coordinates by noting that $\mathbf{r_0} = \mathbf{p}/p$ and then expressing $\mathbf{p}$ in rectangular coordinates. To do this, let Ω be the depression angle of $\mathbf{p}$ from the xy-plane (the horizontal plane) with Ω positive downward and let α be the direction of $\mathbf{p}$ relative to the electrometer axis direction. Then the electric dipole moment $\mathbf{p} = p \cdot (\mathbf{h} \cdot \cos\Omega - \mathbf{k} \cdot \sin\Omega)$ in terms of the unit vector $\mathbf{k}$ and the unit vector $\mathbf{h} = \mathbf{i} \cdot \sin\alpha + \mathbf{j} \cdot \cos\Omega$ which has the direction of the horizontal component of $\mathbf{p}$ relative to the y-axis. Therefore, the unit vector in the dipole direction $\mathbf{r_0} = \mathbf{i} \cdot (\cos\Omega \cdot \sin\alpha) + \mathbf{j} \cdot (\cos\Omega \cdot \cos\alpha) - \mathbf{k} \cdot \sin\Omega$ and

$$E_s = (c \cdot p/r^3) \cdot [(3/r) \cdot \cos\theta \cdot (y \cdot \cos\Phi - z \cdot \sin\Phi) - (\cos\Phi \cdot \cos\Omega \cdot \cos\alpha + \sin\Phi \cdot \sin\Omega)]$$

where $\cos\theta = (1/r) \cdot (x \cdot \cos\Omega \cdot \sin\alpha + y \cdot \cos\Omega \cdot \cos\alpha - z \cdot \sin\Omega)$ since $\cos\theta = \mathbf{r} \bullet \mathbf{r_0}$. As can be seen from this expression, for a constant dipole moment magnitude and direction and a constant electrometer direction, the electrometer signal is a function of the rectangular coordinates of the location of the electrometer relative to the dipole. In the encounter models, both of these conditions are satisfied.

Appendix 4. The Anderson Formulation

In the encounter models, the electric signal E_s at a sample point in a straight line encounter can be represented in a form described by Anderson in Reference 4. For convenience, the Anderson formulation is used in the program to determine values for E_s. It can be developed as follows: In Figure 2, the primed rectangular coordinate system is superimposed on the rectangular coordinate system of Figure 1 so that the origin is coincident with the origin of that system. An electrometer is in a straight line encounter with an electric dipole that is located at the origin of the combined system. The combined system moves with the electric dipole with the x'-axis oriented so that it is parallel to and in the direction of the electrometer's track relative to the dipole and the z'-axis oriented so that it is directed toward and passes through the CPA on that track. In the primed coordinate system, let l, m, and n be the direction cosines of the dipole moment **p** and l_1, m_1 and n_1 be the direction cosines of the unit vector **e**. Then, the unit vector $\mathbf{r}_0 = \mathbf{i}'\cdot l + \mathbf{j}'\cdot n + \mathbf{k}'\cdot m$. And, for points on the relative track, $x' = s'$, $y' = 0$ and $z' = R$ where R is the slant range of the dipole at CPA and s' is the electrometer's algebraic distance from CPA on the relative track. (It is negative for points before CPA and positive for points after CPA.) This implies that the unit vector $\mathbf{r} = \mathbf{i}'\cdot(s'/r) + \mathbf{k}'\cdot(R/r)$. From Appendix 3, the field is $\mathbf{E}_d = (c\cdot p/r^3)\cdot(3\cdot\mathbf{r}\cdot\cos\theta - \mathbf{r}_0)$. With **r** and $\mathbf{r}_0$ expressed in terms of the primed unit vectors and

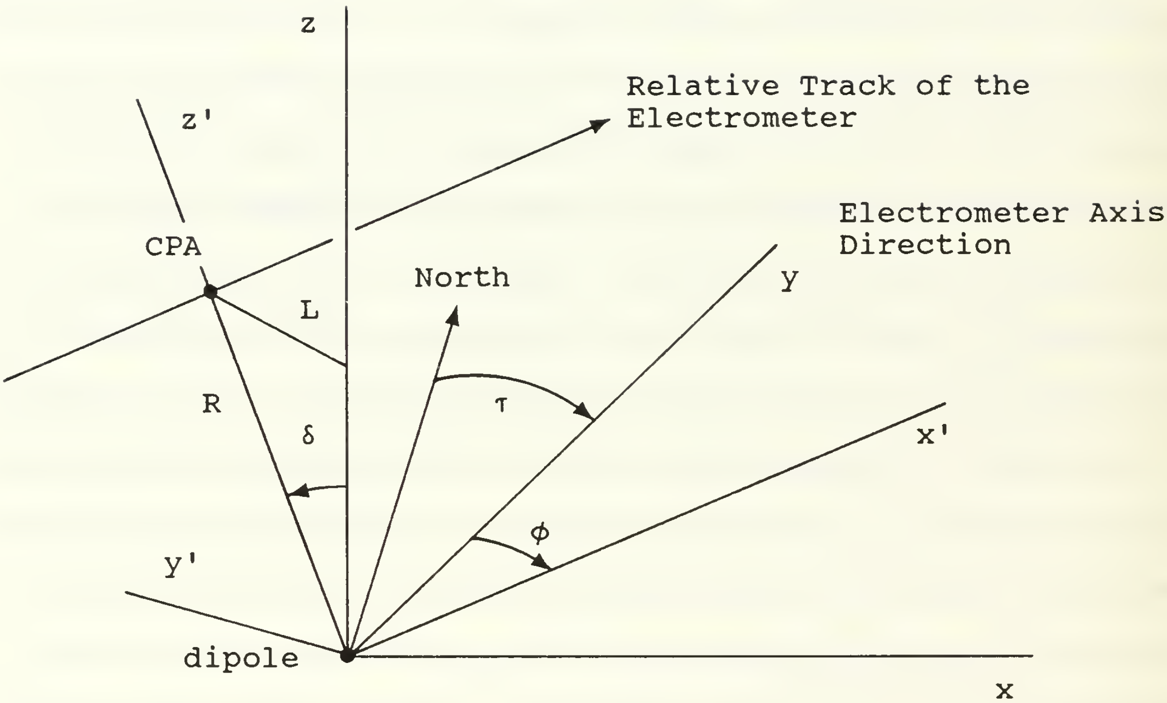

Figure 2. The dipole is at the origin in the unprimed and primed coordinate systems. In the primed coordinate system, the CPA is at $(0, 0, R)$ and, for a time t during the encounter, the electrometer is at $[s'(t), 0, R]$.

$\cos\theta = \mathbf{r}\cdot\mathbf{r}_0 = l\cdot(s'/r) + n\cdot(R/r)$, this becomes

$$\mathbf{E}_d = (c\cdot p/r^3)\cdot[(3/r^2)\cdot(l\cdot s' + n\cdot R)\cdot(\mathbf{i}'\cdot s' + \mathbf{k}'\cdot R)$$
$$- (\mathbf{i}'\cdot l + \mathbf{j}'\cdot m + \mathbf{k}'\cdot n)].$$

Then, since $\mathbf{e} = \mathbf{i}'\cdot l_1 + \mathbf{j}'\cdot m_1 + \mathbf{k}'\cdot n_1$ and $E_s = \mathbf{e}\cdot\mathbf{E}_d$,

$$E_s = (c\cdot p/r^3)\cdot[(2\cdot l\cdot l_1 - m\cdot m_1 - n\cdot n_1)\cdot(s')^2$$
$$+ 3\cdot(n\cdot l_1 + l\cdot n_1)\cdot s'\cdot R + (2\cdot n\cdot n_1 - l\cdot l_1 - m\cdot m_1)\cdot R^2].$$

The quantities

$$A_2 = 2\cdot l\cdot l_1 - m\cdot m_1 - n\cdot n_1$$
$$A_1 = 3\cdot(n\cdot l_1 + l\cdot n_1)$$
$$A_0 = 2\cdot n\cdot n_1 - l\cdot l_1 - m\cdot m_1,$$

are Anderson like coefficients. With $r = [(s')^2 + R^2]^{\frac{1}{2}}$ and $\beta = s'/R$, E_s can now be expressed as follows:

$$E_s(\beta) = (c\cdot p/R^3)\cdot\Sigma\, A_j\cdot F_j(\beta)$$

where the $F_j(\beta) = \beta^j/(1 + \beta^2)^{5/2}$ for $j = 0, 1, 2$ are Anderson like functions. This is the Anderson formulation.

To relate the Anderson formulation for E_s to the formulation for E_s in Appendix 3, first note that

$$l = \mathbf{r}_0\cdot\mathbf{i}'$$
$$m = \mathbf{r}_0\cdot\mathbf{j}'$$
$$n = \mathbf{r}_0\cdot\mathbf{k}'$$

and

$$l_1 = \mathbf{e}\cdot\mathbf{i}'$$
$$m_1 = \mathbf{e}\cdot\mathbf{j}'$$
$$n_1 = \mathbf{e}\cdot\mathbf{k}'.$$

Then express r_0, e and the unit vectors i', j' and k' in terms of the unit vectors i, j and k and take the indicated dot products. From Appendix 3,

$$\mathbf{r}_0 = \mathbf{i}\cdot(\cos\Omega\cdot\sin\alpha) + \mathbf{j}\cdot(\cos\Omega\cdot\cos\alpha) - \mathbf{k}\cdot\sin\Omega$$

and

$$\mathbf{e} = \mathbf{j}\cdot\cos\Phi + \mathbf{k}\cdot\sin\Phi.$$

To express i', j' and k' in terms of the unit vectors i, j and k, note that the unprimed coordinate system can be transformed to the primed coordinate system by two rotations that are defined as follows: First, rotate a coordinate system that is coincident with the unprimed coordinate system about the z-axis so that its x-axis is parallel to and in the direction of the relative track. With positive values clockwise (left hand rule), this is a rotation through the angle $(\phi - \pi/2)$ where as shown in Figure 2, the direction of the electrometer's track relative to the dipole is $\tau + \phi$. Next, rotate this system about its x-axis through an angle δ with positive values counterclockwise (right hand rule) so that the positive z-axis passes through the CPA. The angle δ is related to the vertical separation z_0 of the electrometer and the dipole and the algebraic encounter lateral range L that is positive if the dipole is to the left of the relative track. With these sign definitions: $L = R\cdot\sin\delta$ and $z_0 = R\cdot\cos\delta$. After the rotation, the auxiliary coordinate system is coincident with the primed coordinate system.

These transformations can be described in terms of matrix equations as follows: Let (x'',y'',z'') be the coordinates of a point in the coordinate system that is coincident with the auxiliary coordinate system after the first rotation. Then the transformation from the unprimed coordinate system to this double primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} = \begin{vmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} .$$

And the transformation from the double primed coordinate system to the primed coordinate system is described by the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix} .$$

Taking the product of the rotation matrices in the indicated order yields the matrix equation

$$\begin{vmatrix} x' \\ y' \\ z' \end{vmatrix} = \begin{vmatrix} \sin\phi & \cos\phi & 0 \\ -\cos\delta\cos\phi & \cos\delta\sin\phi & \sin\delta \\ \sin\delta\cos\phi & -\sin\delta\sin\phi & \cos\delta \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

which defines the transformation from the unprimed to the primed coordinate system. The unprimed vector components of the unit vector i' can be found by transforming the coordinates (1,0,0) in the primed system to their corresponding coordinates in the unprimed system with the inverse of this matrix and then repeating this process for (0,1,0) and (0,0,1) in order to find the unprimed unit vector components of **j'** and **k'**. However, since the inverse transformation matrix is the transpose of this matrix, the elements of the row that corresponds to a primed unit vector are the magnitudes of the unprimed vectors that are its components. Consequently:

$$\mathbf{i}' = \mathbf{i}\cdot\sin\phi + \mathbf{j}\cdot\cos\phi$$

$$\mathbf{j}' = -\mathbf{i}\cdot\cos\delta\,\cos\phi + \mathbf{j}\cdot\cos\delta\,\sin\phi + \mathbf{k}\cdot\sin\delta$$

$$\mathbf{k}' = \mathbf{i}\cdot\sin\delta\,\cos\phi - \mathbf{j}\cdot\sin\delta\,\sin\phi + \mathbf{k}\cdot\cos\delta.$$

Then, taking the dot products between $\mathbf{r}$, $\mathbf{r}_0$ and these three unit vectors as indicate above gives:

$$l = \cos\Omega\cdot\cos(\phi - \alpha)$$

$$m = \cos\delta\cdot\cos\Omega\cdot\sin(\phi - \alpha) - \sin\delta\cdot\sin\Omega$$

$$n = -\sin\delta\cdot\cos\Omega\cdot\sin(\phi - \alpha) - \cos\delta\cdot\sin\Omega$$

and

$$l_1 = \cos\Phi\cdot\cos\phi$$

$$m_1 = \cos\delta\cdot\cos\Phi\cdot\sin\phi - \sin\delta\cdot\sin\Phi$$

$$n_1 = -\sin\delta\cdot\cos\Phi\cdot\sin\phi - \cos\delta\cdot\sin\Phi.$$

These are the relations that are used in the program to determine values for the Anderson like coefficients.

Appendix 5. The Encounter Equations of Motion

In the double primed coordinate system that is defined in Appendix 4, the equations of motion of an electrometer relative to a submarine (dipole) are:

$$x''(t) = s'(t)$$

$$y''(t) = -L$$

$$z''(t) = z_0.$$

where L is the algebraic encounter lateral range that is defined in Appendix 4, z_0 is the vertical separation between the electrometer and the submarine and $s'(t)$ is the distance of the electrometer from the CPA on the relative track. With w the speed of the electrometer relative to the submarine and t a relative time parameter, $s'(t) = w \cdot t$. These equations can be considered to be the ones used in the program to describe the motion of a electrometer relative to a submarine. There, t is determined by $t = [j - (m-1)/2] \cdot \delta t$ where the index $j = 1, 2, \cdots, m$ and δt, a time step, is the time between samples. Note, when $t = 0$, the electrometer is at the CPA.

In the coordinate system of Figure 1 in a straight line encounter as defined in the encounter models, the equations of motion of a electrometer relative to a submarine can be written as follows:

$$x(t) = s'(t) \cdot \sin \phi \ + \ L \cdot \cos \phi$$

$$y(t) = s'(t) \cdot \cos \phi \ - \ L \cdot \sin \phi$$

$$z(t) = z_0,$$

since the transformation from the double primed coordinate system to the primed coordinate system is determined by the matrix equation

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \sin\phi & -\cos\phi & 0 \\ \cos\phi & \sin\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x'' \\ y'' \\ z'' \end{vmatrix},$$

$x'' = s'(t)$ and $y'' = -L$. The above equations and the expression for E_s in Appendix 3 could have been used in the program to evaluate the electric signal. In particular, with these equations of motion and with the two relations $L = R\cdot\sin\delta$ and $z_0 = R\cdot\cos\delta$, the expression for H_s in Appendix 3 can be written in terms of Ω, α, Φ, ϕ, δ and R so that it is identical in appearance to the Anderson formulation for E_s in terms of these quantities. The definition of δ in Appendix 4 in terms of a counterclockwise rotation results in a definition of the algebraic lateral range that is consistent with some that have been used elsewhere.

In the program, the relation $\mathbf{w} = \mathbf{v} - \mathbf{u}$ is the basis for determining the relative speed w. In this relation, $\mathbf{v}$ is the velocity of the electrometer, $\mathbf{u}$ is the velocity of the submarine (dipole) and $\mathbf{w}$ is the velocity of the electrometer relative to the submarine. This relation implies the following equations: $w_x = v_x - u_x$, $w_y = v_y - u_y$ and $w_z = v_z - u_z$ where the coordinates x, y and z refer to a fixed coordinate system with the same orientation as that of Figure 1. In the encounters of the models, $v_x = v \cdot \sin(\sigma - \tau)$, $v_y = v \cdot \cos(\sigma - \tau)$ and $v_z = 0$ where σ is the course of the electrometer and v is the speed of the electrometer. And, $u_x = u \cdot \sin(\beta - \tau)$, $u_y = u \cdot \cos(\beta - \tau)$ and $u_z = 0$ where β is the course and u is the speed of the submarine. The angle ϕ and the relative speed of the electrometer are defined by $w_x = w \cdot \sin\phi$ and $w_y = w \cdot \cos\phi$. In the program, ϕ and w are determined with a rectangular to polar conversion routine where $w = (w_x^2 + w_y^2)^{\frac{1}{2}}$ and ϕ is determined by $\sin^{-1}(w_x/w)$ and $\cos^{-1}(w_y/w)$.

Appendix 6. The Submarine Electric Dipole

In the encounter models, the magnitude and direction of a submarine's dipole moment p are determined by first determining its components in the rectangular coordinate system of Figure 1. In that coordinate system,

$$p_x = p_{LP} \cdot \sin(\beta - \tau) + p_{TP} \cdot \cos(\beta - \tau)$$

$$p_y = p_{LP} \cdot \cos(\beta - \tau) - p_{TP} \cdot \sin(\beta - \tau)$$

$$p_z = -p_{VP}$$

where $\beta - \tau$ is the submarine's course relative to the electrometer axis direction and p_{LP}, p_{TP} and p_{VP} are the longitudinal, transverse and vertical electric dipole moments of the submarine. These relations are based on the following sign convention:

p_L is positive when p_L is directed from stern to bow.

p_T is positive when p_T is directed from port to starboard.

p_V is positive when p_V is directed downward.

The electric dipole moments are input parameters in the program.

Appendix 7. An Alternative Encounter Model

The alternative model that is referred to in Section III is described in more detail in this appendix. In the model, the detection range is the slant range R at the CPA for an encounter with a specified detection probability (usually .5) is defined by:

$$R = [c \cdot p/H_S]^{1/3}$$

where c and p are defined in Appendix 3 and E_S represents a minimum detectable average electric signal that is defined by:

$$E_S = (ORF) \cdot N_E$$

where ORF is a signal-to-noise ratio called the operator recognition factor and N_E represents the electric noise. Combining these two relations gives:

$$R = \{c \cdot p/[(ORF) \cdot N_E]\}^{1/3}.$$

The value for ORF depends both on the specified detection probability and on a specified or implied false alarm probability.

The Anderson formulation is consistent with these relations in an approximate sense if the average electric signal E is defined as a root mean square value such that $E = (c \cdot p/R^3) \cdot k$ where k is an encounter parameter defined by:

$$k = \{\Sigma\ [\Sigma\ A_j \cdot F_j(\beta_i)]^2\}^{\frac{1}{2}}$$

with the first sum index $i = 1, 2, \cdots, m$ and the second sum index $j = 0, 1, 2$. For a particular encounter geometry, k is constant and this suggests that the two encounter models could be used to determine an average value for k for an encounter region based on average submarine electric characteristics. Values for both k and E are generated by the program and such values can give an indication of the magnitude of the differences in detection range estimates that are based on this model and those that are based on either of the other two encounter models.

Appendix 8. An Example of the Program Output

The program is designed to generate the following quantities: encounter parameter values, lateral range function values for the crosscorrelation encounter model and for the square law encounter model, average electric signal values, slant range at CPA values, encounter parameter values and electric signal values. These values can be saved as a program data file and/or they can be printed.

An example of the program's printed output is listed in Table 1, Table 2 and Table 3. Figure 3 is a plot of the lateral range function values (a lateral range curve) for the square law detector that are listed in Table 2. Figure 4 is a plot of the electric signal values that are listed in Table 3.

A second example of the program's printed output is listed in Table 4, Table 5 and Table 6. Figure 5 is a plot of the lateral range function values for the square law detector that are listed in Table 5. Figure 6 is a plot of the electric signal values that are listed in Table 6.

program file name	EAD.BAS
program data file name	data.ead
electric data file name	data.ele
processing data file name	data.prc
kinematic data file name	data.kim
combined electric, processing & kinematic data file name	data.epk
electrometer axis direction (decimal degrees)	335
electrometer axis depression angle (decimal degrees)	0
encounter medium conductivity (mhos/meter)	4.2
electric longitudinal moment (ampere-meters)	25
electric transverse moment (ampere-meters)	0
electric vertical moment (ampere-meters)	0
sampling period (seconds)	10
integration time (seconds)	400
adjusted integration time (seconds)	410
number of samples per encounter	41
electrometer course (decimal degrees)	0
electrometer speed (knots)	0
electrometer altitude (meters)	0
target course (decimal degrees)	57
target speed (knots)	10
target depth (meters)	-100
electrometer relative course (decimal degrees)	237
electrometer relative speed (knots)	10
electrometer-target vertical separation (meters)	-100
electric dipole moment (ampere-meters)	25
dipole moment azimuth (decimal degrees)	57.00003
dipole moment depression angle (decimal degrees)	0
distance between samples on the relative track (meters)	51.44445
false alarm rate (false alarms per hour)	.15
false alarm probability	1.666667E-02
electric noise (volts/meter)	3.5E-08
maximum lateral range (meters)	500
lateral range step (meters)	20
number of lateral range function values	51

Table 1. An example of an encounter parameter values printout.

data.ead lateral range function values

L meters	p(cc)	p(sl)	E volts/meter	R meters	k
-500	.0359629	1.495071E-02	5.95766E-09	509.902	3.221905
-480	3.869068E-02	1.506978E-02	6.713175E-09	490.306	3.156311
-460	4.201478E-02	1.522109E-02	7.589571E-09	470.7441	3.088742
-440	4.611728E-02	.0154154	8.597962E-09	451.2206	3.019128
-420	5.125207E-02	1.566772E-02	9.758901E-09	431.7407	2.947377
-400	.0577791	1.599973E-02	1.114912E-08	412.3106	2.873376
-380	6.621807E-02	1.644293E-02	1.28209E-08	392.9377	2.796981
-360	7.733236E-02	1.704431E-02	1.492985E-08	373.6308	2.718022
-340	9.226168E-02	1.787643E-02	1.743602E-08	354.4009	2.636287
-320	.1127298	1.905402E-02	2.041433E-08	335.2611	2.551514
-300	.1413591	2.076598E-02	2.415458E-08	316.2278	2.463387
-280	.1821076	2.333736E-02	2.882961E-08	297.3214	2.371516
-260	.2407389	2.735578E-02	3.454627E-08	278.5678	2.275416
-240	.3248692	3.394424E-02	4.222557E-08	260	2.174487
-220	.4421915	4.537476E-02	5.17294E-08	241.6609	2.067975
-200	.5940236	6.647302E-02	6.409887E-08	223.6068	1.954923
-180	.7625715	.107666	8.065302E-08	205.9126	1.834113
-160	.9040051	.1900176	1.010635E-07	188.6796	1.703969
-140	.9785036	.3451494	1.25225E-07	172.0465	1.562464
-120	.9979309	.5807582	1.576369E-07	156.205	1.406982
-100	.9999207	.8128465	1.953249E-07	141.4214	1.234252
-80	.9999976	.9348596	2.313523E-07	128.0625	1.04051
-60	.9999995	.9610708	2.5821E-07	116.619	.8225375
-40	.9999842	.8823538	2.39062E-07	107.7033	.5807222
-20	.9799458	.3525332	1.489415E-07	101.9804	.3276647
0	.5137237	5.417071E-02	7.803533E-08	100	.1598169
20	.9799458	.3525332	1.489415E-07	101.9804	.3276647
40	.9999842	.8823538	2.39062E-07	107.7033	.5807222
60	.9999995	.9610707	2.5821E-07	116.619	.8225374
80	.9999976	.9348597	2.313523E-07	128.0625	1.04051
100	.9999207	.8128465	1.953249E-07	141.4214	1.234252
120	.9979309	.5807582	1.57637E-07	156.205	1.406983
140	.9785036	.3451494	1.25225E-07	172.0465	1.562464
160	.9040054	.1900176	1.010635E-07	188.6796	1.70397
180	.7625715	.107666	8.065302E-08	205.9126	1.834113
200	.5940236	6.647302E-02	6.409888E-08	223.6068	1.954923
220	.4421915	4.537467E-02	5.172939E-08	241.6609	2.067974
240	.3248694	3.394424E-02	4.222557E-08	260	2.174487
260	.2407389	2.735578E-02	3.454627E-08	278.5678	2.275416
280	.1821077	2.333736E-02	2.882961E-08	297.3214	2.371516
300	.1413592	2.076598E-02	2.415459E-08	316.2278	2.463388
320	.1127298	1.905402E-02	2.041433E-08	335.2611	2.551514
340	9.226175E-02	1.787643E-02	1.743602E-08	354.4009	2.636287
360	7.733236E-02	1.704431E-02	1.492985E-08	373.6308	2.718022
380	6.621807E-02	1.644293E-02	1.28209E-08	392.9377	2.796981
400	5.777909E-02	1.599973E-02	1.114912E-08	412.3106	2.873375
420	5.125207E-02	1.566772E-02	9.758901E-09	431.7407	2.947377
440	4.611728E-02	.0154154	8.597962E-09	451.2206	3.019128
460	4.201478E-02	1.522109E-02	7.589571E-09	470.7441	3.088743
480	3.869068E-02	1.506978E-02	6.713175E-09	490.306	3.156311
500	.0359629	1.495071E-02	5.95766E-09	509.902	3.221906

Table 2. An example of a lateral range function values printout. The heading for the crosscorrelation values is p(cc) and the heading for the square law values is p(sl).

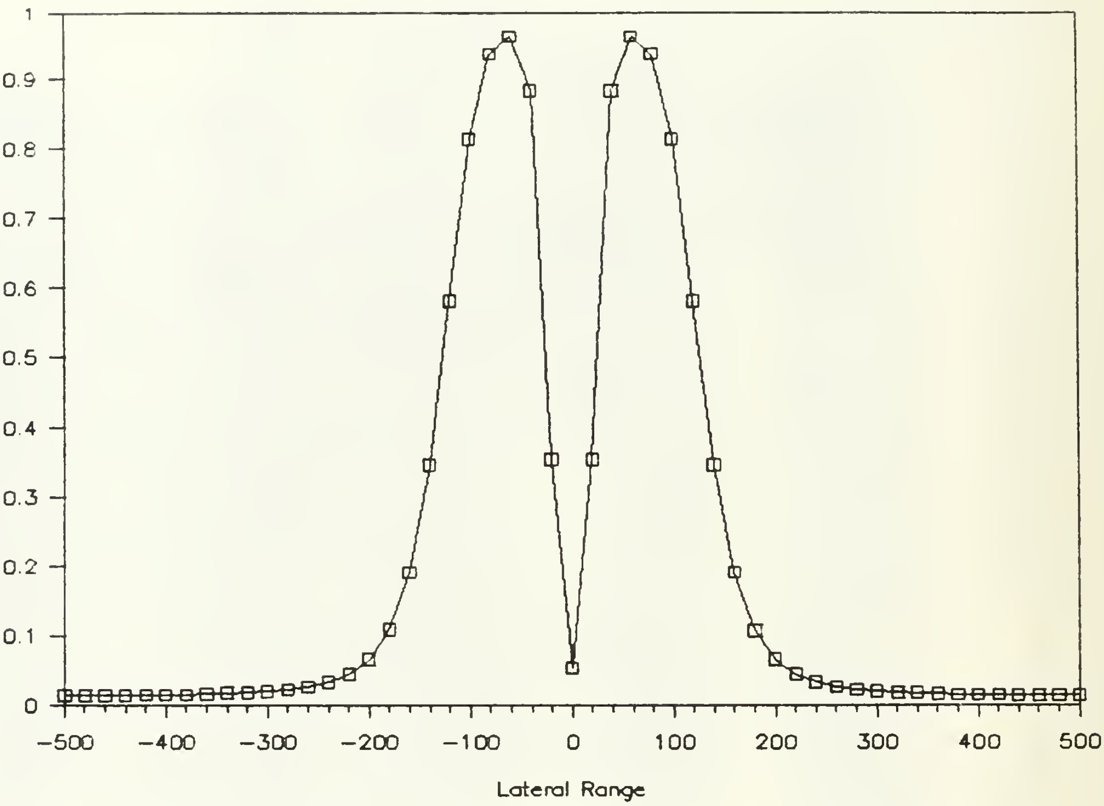

Figure 3. A plot of the square law lateral range function values that are listed in Table 2. The horizontal axis is encounter horizontal range at CPA in meters. The vertical axis is encounter detection probability.

data.ead electric signal values
for a lateral range of 40 meters

relative CPA distance s in meters	electric signal Es in volts/meter
-1028.889	1.660273E-10
-977.4446	1.959909E-10
-926.0001	2.335016E-10
-874.5557	2.810548E-10
-823.1112	3.421893E-10
-771.6667	4.220248E-10
-720.2223	5.28136E-10
-668.7778	6.720149E-10
-617.3334	8.715932E-10
-565.889	1.155758E-09
-514.4445	1.572761E-09
-463.0001	2.206581E-09
-411.5556	3.210406E-09
-360.1111	4.878809E-09
-308.6667	7.81198E-09
-257.2223	1.330549E-08
-205.7778	2.426627E-08
-154.3333	4.685768E-08
-102.8889	8.75983E-08
-51.44445	1.023685E-07
0	-5.276525E-08
51.44445	-1.366934E-07
102.8889	-7.038444E-08
154.3333	-2.673259E-08
205.7778	-1.000855E-08
257.2223	-3.866346E-09
308.6667	-1.493584E-09
360.1111	-5.247395E-10
411.5556	-1.147233E-10
463.0001	5.922923E-11
514.4445	1.289336E-10
565.889	1.514067E-10
617.3334	1.524577E-10
668.7778	1.44157E-10
720.2223	1.32254E-10
771.6667	1.194879E-10
823.1112	1.071295E-10
874.5557	9.572235E-11
926.0001	8.545079E-11
977.4446	7.632433E-11
1028.889	6.827254E-11

Table 3. An example of an electric signal values printout.

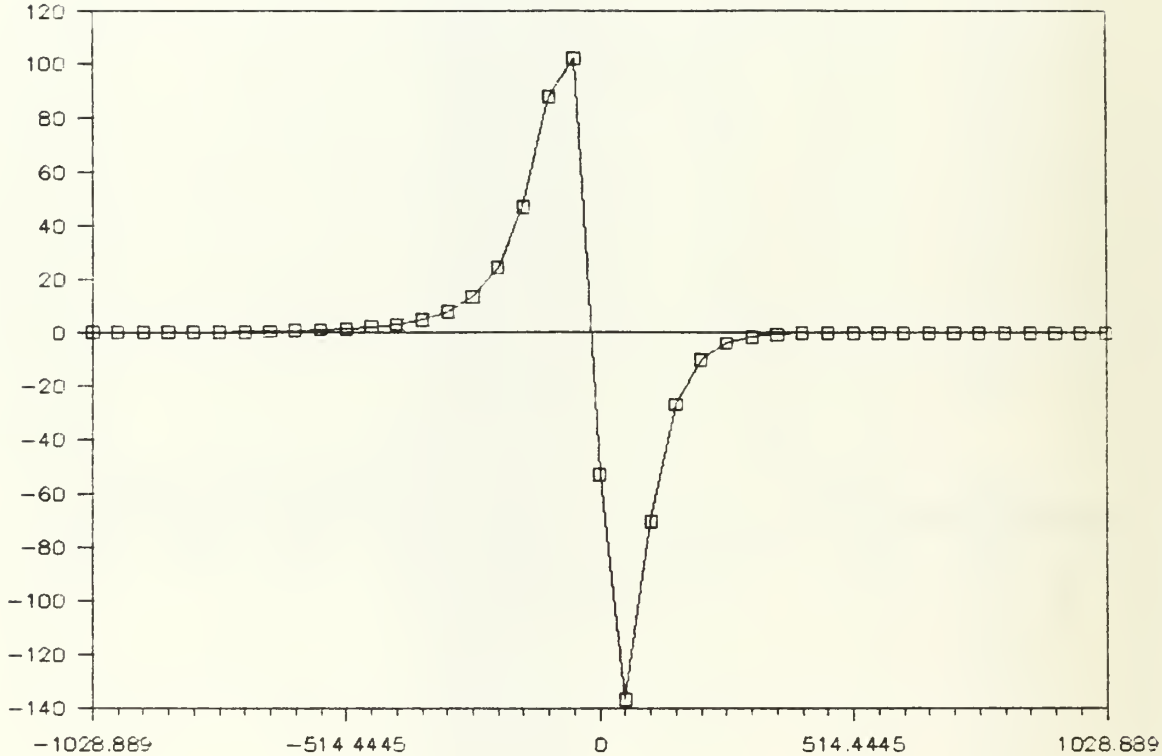

Figure 4. A plot of the electric signal values listed in Table 3. The horizontal axis is relative CPA distance in meters. The vertical axis is electric signal in nanovolts per meter.

program file name	EAD.BAS
program data file name	data1.ead
electric data file name	data.ele
processing data file name	data.prc
kinematic data file name	data1.kim
combined electric, processing & kinematic data file name	data1.epk
electrometer axis direction (decimal degrees)	335
electrometer axis depression angle (decimal degrees)	0
encounter medium conductivity (mhos/meter)	4.2
electric longitudinal moment (ampere-meters)	25
electric transverse moment (ampere-meters)	0
electric vertical moment (ampere-meters)	0
sampling period (seconds)	10
integration time (seconds)	400
adjusted integration time (seconds)	410
number of samples per encounter	41
electrometer course (decimal degrees)	0
electrometer speed (knots)	0
electrometer altitude (meters)	0
target course (decimal degrees)	170
target speed (knots)	10
target depth (meters)	-100
electrometer relative course (decimal degrees)	350
electrometer relative speed (knots)	10
electrometer-target vertical separation (meters)	-100
electric dipole moment (ampere-meters)	25
dipole moment azimuth (decimal degrees)	170
dipole moment depression angle (decimal degrees)	0
distance between samples on the relative track (meters)	51.44444
false alarm rate (false alarms per hour)	.15
false alarm probability	1.666667E-02
electric noise (volts/meter)	3.5E-08
maximum lateral range (meters)	500
lateral range step (meters)	20
number of lateral range function values	51

Table 4. An example of an encounter parameter values printout.

data1.ead lateral range function values

L meters	p(cc)	p(sl)	E volts/meter	R meters	k
-500	3.110245E-02	1.475347E-02	4.65646E-09	509.9019	2.582769
-480	3.308339E-02	1.483123E-02	5.235783E-09	490.306	2.536152
-460	3.548338E-02	1.493034E-02	5.903325E-09	470.7441	2.488133
-440	3.842717E-02	1.505803E-02	6.703113E-09	451.2206	2.438668
-420	4.208816E-02	1.522449E-02	7.637841E-09	431.7407	2.387708
-400	4.671227E-02	1.544411E-02	8.748915E-09	412.3106	2.335211
-380	5.265528E-02	1.573816E-02	1.009386E-08	392.9377	2.28113
-360	6.044344E-02	1.613816E-02	1.168881E-08	373.6308	2.225419
-340	.0708726	1.669256E-02	1.366401E-08	354.4009	2.168032
-320	8.517325E-02	1.747827E-02	1.606643E-08	335.2611	2.108919
-300	.1052861	1.862119E-02	1.899775E-08	316.2278	2.048029
-280	.1343149	2.033773E-02	2.283886E-08	297.3214	1.985313
-260	.1772254	2.302039E-02	2.769995E-08	278.5678	1.92072
-240	.2417317	2.742749E-02	3.402688E-08	260	1.854206
-220	.3386843	.0351379	4.239666E-08	241.6609	1.785746
-200	.4792239	4.971678E-02	5.322737E-08	223.6068	1.715341
-180	.662185	7.983379E-02	6.817646E-08	205.9126	1.643053
-160	.8491701	.1472224	8.849707E-08	188.6796	1.569052
-140	.9671919	.300248	1.156304E-07	172.0465	1.4937
-120	.9981669	.5916718	1.542097E-07	156.205	1.417713
-100	.9999912	.9016253	2.073466E-07	141.4214	1.34246
-80	1	.9966924	2.759074E-07	128.0625	1.270434
-60	1	.9999966	3.585535E-07	116.619	1.205756
-40	1	1	4.465976E-07	107.7033	1.154001
-20	1	1	5.20507E-07	101.9804	1.120663
0	1	1	5.416029E-07	100	1.109207
20	1	1	5.20507E-07	101.9804	1.120663
40	1	1	4.465976E-07	107.7033	1.154001
60	1	.9999966	3.585535E-07	116.619	1.205757
80	1	.9966924	2.759074E-07	128.0625	1.270434
100	.9999912	.9016253	2.073466E-07	141.4214	1.34246
120	.9981669	.5916718	1.542097E-07	156.205	1.417713
140	.9671919	.300248	1.156304E-07	172.0465	1.493699
160	.8491701	.1472224	8.849707E-08	188.6796	1.569052
180	.662185	7.983379E-02	6.817645E-08	205.9126	1.643053
200	.4792239	4.971678E-02	5.322737E-08	223.6068	1.715341
220	.3386843	.0351379	4.239666E-08	241.6609	1.785746
240	.2417317	2.742749E-02	3.402688E-08	260	1.854206
260	.1772254	2.302039E-02	2.769995E-08	278.5678	1.920719
280	.1343149	2.033773E-02	2.283886E-08	297.3214	1.985313
300	.1052861	1.862119E-02	1.899775E-08	316.2278	2.04803
320	8.517325E-02	1.747827E-02	1.606643E-08	335.2611	2.108919
340	7.087261E-02	1.669256E-02	1.366401E-08	354.4009	2.168032
360	6.044344E-02	1.613816E-02	1.168881E-08	373.6308	2.225419
380	5.265528E-02	1.573816E-02	1.009386E-08	392.9377	2.28113
400	4.671226E-02	1.544411E-02	8.748914E-09	412.3106	2.335211
420	4.208816E-02	1.522449E-02	7.637841E-09	431.7407	2.387708
440	3.842716E-02	1.505803E-02	6.703113E-09	451.2206	2.438667
460	3.548338E-02	1.493034E-02	5.903325E-09	470.7441	2.488133
480	3.308338E-02	1.483123E-02	5.235783E-09	490.306	2.536152
500	3.110243E-02	1.475347E-02	4.65646E-09	509.9019	2.582769

Table 5. An example of a lateral range function values printout. The heading for the crosscorrelation values is p(cc) and the heading for the square law values is p(sl).

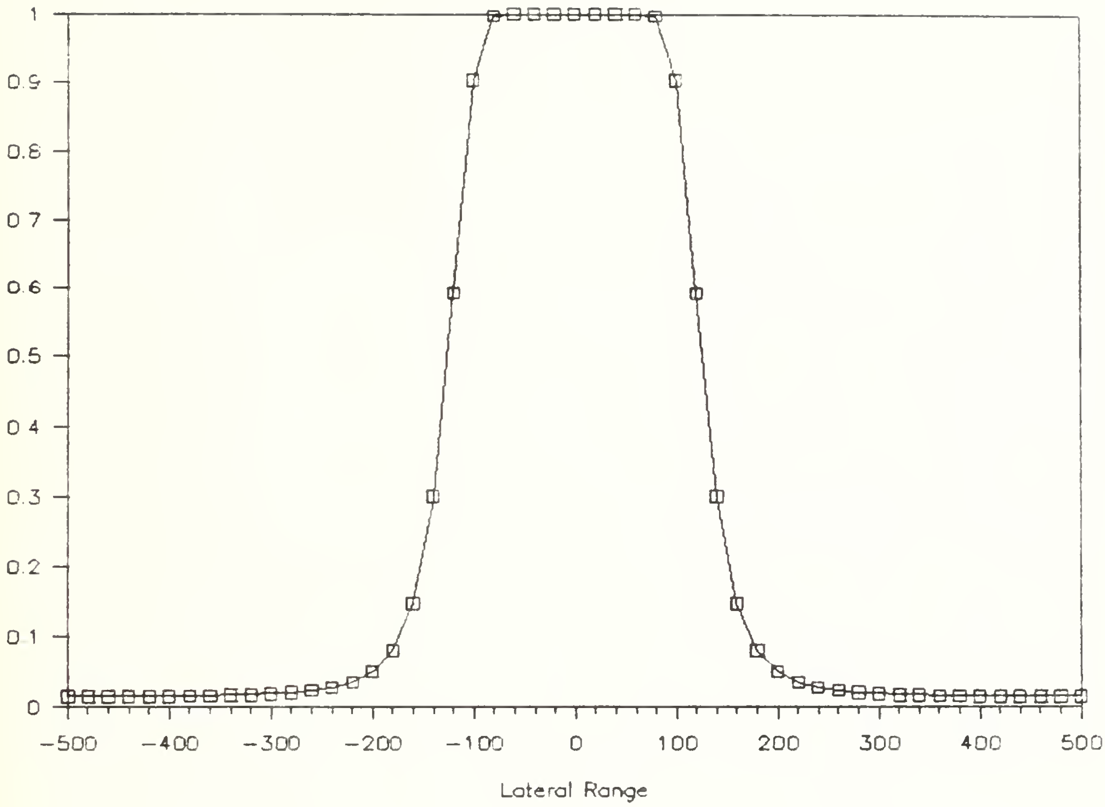

Figure 5. A plot of the square law lateral range function values that are listed in Table 5. The horizontal axis is encounter horizontal range at CPA in meters. The vertical axis is encounter detection probability.

datal.ead electric signal values
for a lateral range of 40 meters

relative CPA distance s in meters	electric signal Es in volts/meter
-1028.889	-8.258498E-10
-977.4443	-9.606357E-10
-925.9999	-1.126187E-09
-874.5555	-1.331725E-09
-823.111	-1.589959E-09
-771.6666	-1.918711E-09
-720.2222	-2.343442E-09
-668.7777	-2.901288E-09
-617.3333	-3.647673E-09
-565.8889	-4.667418E-09
-514.4444	-6.093956E-09
-462.9999	-8.143509E-09
-411.5555	-1.117728E-08
-360.1111	-1.581578E-08
-308.6666	-2.314237E-08
-257.2222	-3.500006E-08
-205.7778	-5.39567E-08
-154.3333	-7.945557E-08
-102.8889	-8.038128E-08
-51.44444	8.787511E-08
0	3.662163E-07
51.44444	1.503571E-07
102.8889	-3.909039E-08
154.3333	-6.022178E-08
205.7778	-4.499852E-08
257.2222	-3.051197E-08
308.6666	-2.071024E-08
360.1111	-1.44035E-08
411.5555	-1.030821E-08
462.9999	-7.582271E-09
514.4444	-5.716593E-09
565.8889	-4.404917E-09
617.3333	-3.459717E-09
668.7777	-2.763325E-09
720.2222	-2.239973E-09
771.6666	-1.839639E-09
823.111	-1.528523E-09
874.5555	-1.283285E-09
925.9999	-1.087492E-09
977.4443	-9.293593E-10
1028.889	-8.003003E-10

Table 6. An example of an electric signal values printout.

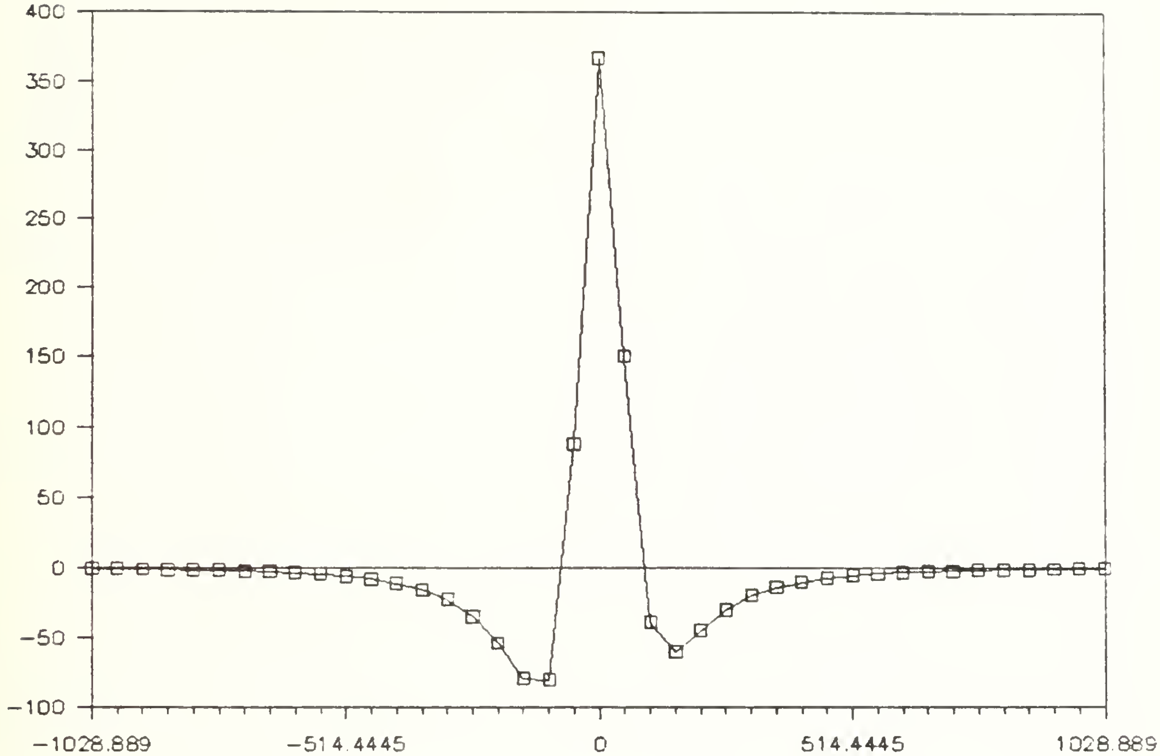

Figure 6. A plot of electric signal values listed in Table 6. The horizontal axis is relative CPA distance in meters. The vertical axis is electric signal in nanovolts per meter.

Appendix 9. A Program Listing

```
10 CLS : CLEAR
20 PRINT "Electric Anomaly Detection (EAD) Lateral Range Function Program"
30 DIM X0(70), R0(70), E(70), ES(70, 150), K(70), PDCC(70), PDSL(70)
40 PI = 3.141592654#: CON = 2 * PI / 360: KON = 1852 / 3600: N$ = "EAD.BAS"
50 Q0 = .2316419: Q1 = .31938153#: Q2 = -.356563782#: Q3 = 1.781477937#
60 Q4 = -1.821255978#: Q5 = 1.330274429#
70 I1 = 2.515517: I2 = .802853: I3 = .010328: I4 = 1.432788: I5 = .189269: I6
= .001308
80 PRINT
90 INPUT "generate data or print a program data file (g/p)"; A$
100 IF A$ = "G" OR A$ = "g" THEN GOTO 120
110 IF A$ = "P" OR A$ = "p" THEN GOTO 2700 ELSE GOTO 90
120 PRINT : A$ = "a"
130 INPUT "electric, processing & kinematic data entry by combined file
(y/n)"; A$
140 IF A$ = "Y" OR A$ = "y" THEN GOTO 1740
150 IF A$ = "N" OR A$ = "n" THEN GOTO 160 ELSE GOTO 130
160 INPUT "electric data entry by file/keyboard (f/k)"; A$
170 IF A$ = "F" OR A$ = "f" THEN GOTO 430
180 IF A$ = "K" OR A$ = "k" THEN GOTO 190 ELSE GOTO 160
190 INPUT "electrometer axis direction in decimal degrees"; EAD
200 INPUT "electrometer axis depression in decimal degrees (down +)"; DIP
210 A$ = "a"
220 INPUT "input encounter medium conductivity (y/n)"; A$
230 IF A$ = "Y" OR A$ = "y" THEN GOTO 250
240 IF A$ = "N" OR A$ = "n" THEN GOTO 270 ELSE GOTO 220
250 INPUT "encounter medium conductivity in mohs"; COND
260 GOTO 280
270 COND = 4
280 ELM = 0: ETM = 0: EVM = 0
290 INPUT "longitudinal electric moment in ampere-meters (stern-to-bow +)";
ELM
300 INPUT "transverse electric moment in ampere-meters (port-to-starboard +)";
ETM
310 INPUT "vertical electric moment in ampere-meters (downward +)"; EVM
320 A$ = "a"
330 INPUT "generate an electric data file (y/n)"; A$
340 IF A$ = "Y" OR A$ = "y" THEN GOTO 360
350 IF A$ = "N" OR A$ = "n" THEN GOTO 490 ELSE GOTO 330
360 INPUT "electric data file name"; E$
370 ON ERROR GOTO 380: GOTO 390
380 RESUME 360
390 OPEN "O", #1, E$
400 WRITE #1, EAD, DIP, COND, ELM, ETM, EVM
410 CLOSE
420 GOTO 490
430 INPUT "electric data file name"; E$
440 ON ERROR GOTO 450: GOTO 460
450 RESUME 430
460 OPEN "I", #1, E$
470 INPUT #1, EAD, DIP, COND, ELM, ETM, EVM
```

```
480 CLOSE
490 A$ = "a"
500 INPUT "processing data entry by file/keyboard (f/k)"; A$
510 IF A$ = "F" OR A$ = "f" THEN GOTO 1090
520 IF A$ = "K" OR A$ = "k" THEN GOTO 530 ELSE GOTO 500
530 A$ = "a"
540 INPUT "input sampling period (y/n)"; A$
550 IF A$ = "Y" OR A$ = "y" THEN GOTO 570
560 IF A$ = "N" OR A$ = "n" THEN GOTO 600 ELSE GOTO 540
570 INPUT "sampling period in seconds"; DT
580 IF DT <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
570
590 GOTO 730
600 A$ = "a"
610 INPUT "input maximum electric signal frequency (y/n)"; A$
620 IF A$ = "Y" OR A$ = "y" THEN GOTO 640
630 IF A$ = "N" OR A$ = "n" THEN GOTO 670 ELSE GOTO 610
640 INPUT "maximum electric signal frequency in Hertz"; MAXF
650 IF MAXF <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
640
660 GOTO 720
670 INPUT "minimum target slant range at CPA in meters"; MINR0
680 IF MINR0 <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
670
690 INPUT "maximum electrometer relative speed in knots"; MAXVEK
700 MAXVE = MAXVEK * KON: MAXF = 2 * MAXVE / MINR0
710 IF MAXVM <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
690
720 DT = 1 / (2 * MAXF): REM low pass filter, Nyquist sampling rate
730 A$ = "a"
740 INPUT "input integration time (y/n)"; A$
750 IF A$ = "Y" OR A$ = "y" THEN GOTO 770
760 IF A$ = "N" OR A$ = "n" THEN GOTO 850 ELSE GOTO 740
770 INPUT "integration time in seconds"; IT
780 IF IT >= DT THEN GOTO 810
790 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(DT) + "
seconds": PRINT
800 GOTO 770
810 NS = 2 * INT(IT / DT / 2) + 1: REM adj number of samples per integration
time
820 IF NS <= 151 THEN GOTO 980
830 PRINT : PRINT "IT = " + STR$(IT) + " seconds -  maximum = " + STR$(150 *
DT) + " seconds": PRINT
840 GOTO 770
850 INPUT "maximum target slant range at CPA in meters"; MAXR0
860 IF MAXR0 <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
850
870 INPUT "minimum electrometer relative speed in knots"; MINVEK
880 MINVE = MINVEK * KON
890 IF MINVE <= 0 THEN PRINT : PRINT "must be greater than zero": PRINT : GOTO
870
900 IT = 2 * MAXR0 / MINVE
```

```
910 IF IT >= DT THEN GOTO 940
920 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(DT) + "
seconds": PRINT
930 GOTO 740
940 NS = 2 * INT(IT / DT / 2) + 1: REM adjusted number of samples per
integration time
950 IF NS <= 151 THEN GOTO 980
960 PRINT : PRINT "IT = " + STR$(IT) + " seconds - minimum = " + STR$(150 *
DT) + " seconds": PRINT
970 GOTO 740
980 A$ = "a"
990 INPUT "generate a processing data file (y/n)"; A$
1000 IF A$ = "Y" OR A$ = "y" THEN GOTO 1020
1010 IF A$ = "N" OR A$ = "n" THEN GOTO 1150 ELSE GOTO 990
1020 INPUT "processing data file name"; P$
1030 ON ERROR GOTO 1040: GOTO 1050
1040 RESUME 1020
1050 OPEN "O", #1, P$
1060 WRITE #1, DT, IT, NS
1070 CLOSE
1080 GOTO 1150
1090 INPUT "processing data file name"; P$
1100 ON ERROR GOTO 1110: GOTO 1120
1110 RESUME 1090
1120 OPEN "I", #1, P$
1130 INPUT #1, DT, IT, NS
1140 CLOSE
1150 A$ = "a"
1160 INPUT "kinematic data entry by file/keyboard (f/k)"; A$
1170 IF A$ = "F" OR A$ = "f" THEN GOTO 1440
1180 IF A$ = "K" OR A$ = "k" THEN GOTO 1190 ELSE GOTO 1150
1190 INPUT "electrometer course in decimal degrees (0 if stationary)"; CE
1200 CER = CE * CON: REM electrometer course in radians
1210 INPUT "elecrometer speed in knots"; VEK
1220 INPUT "electrometer altitude in meters (below 0 is -)"; AE
1230 INPUT "target course in decimal degrees"; CT
1240 CTR = CT * CON: REM target course in radians
1250 INPUT "target speed in knots"; VTK
1260 INPUT "target depth in meters (above 0 is -)"; AT
1270 WXK = VEK * SIN(CER) - VTK * SIN(CTR)
1280 WYK = VEK * COS(CER) - VTK * COS(CTR)
1290 Z = AE + AT: REM vertical separation (- for electrometer below target)
1300 X = WXK: Y = WYK: GOSUB 3680
1310 CR = T: C = CR / CON: REM electrometer relative course
1320 WK = R: REM electrometer relative speed
1330 A$ = "a"
1340 INPUT "generate a kinematic data file (y/n)"; A$
1350 IF A$ = "Y" OR A$ = "y" THEN GOTO 1370
1360 IF A$ = "N" OR A$ = "n" THEN GOTO 1510 ELSE GOTO 1340
1370 INPUT "kinematic data file name"; K$
1380 ON ERROR GOTO 1390: GOTO 1400
1390 RESUME 1370
```

```
1400 OPEN "O", #1, K$
1410 WRITE #1, CE, VEK, AE, CT, VTK, AT, Z, C, WK
1420 CLOSE
1430 GOTO 1510
1440 INPUT "kinematic data file name"; K$
1450 ON ERROR GOTO 1460: GOTO 1470
1460 RESUME 1440
1470 OPEN "I", #1, K$
1480 INPUT #1, CE, VEK, AE, CT, VTK, AT, Z, C, WK
1490 CLOSE
1500 CTR = CT * CON: CR = C * CON
1510 EADR = EAD * CON
1520 EMX = ELM * SIN(CTR - EADR) + ETM * COS(CTR - EADR)
1530 EMY = ELM * COS(CTR - EADR) - ETM * SIN(CTR - EADR)
1540 EMZ = -EVM
1550 X = EMX: Y = EMY
1560 GOSUB 3680
1570 OMLR = T: REM dipole azimuth relative to the electrometer axis direction
1580 X = -EMZ: Y = R
1590 GOSUB 3680
1600 EM = R: OMR = T: REM dipole depression angle
1610 OML = OMLR / CON: OM = OMR / CON
1620 A$ = "a"
1630 INPUT "generate a combined electric, processing & kinematic data file
(y/n)"; A$
1640 IF A$ = "Y" OR A$ = "y" THEN GOTO 1660
1650 IF A$ = "N" OR A$ = "n" THEN GOTO 1820 ELSE GOTO 1630
1660 INPUT "combined electric, processing & kinematic data file name"; C$
1670 ON ERROR GOTO 1680: GOTO 1690
1680 RESUME 1660
1690 OPEN "O", #1, C$
1700 WRITE #1, EAD, DIP, COND, ELM, ETM, EVM, DT, IT
1710 WRITE #1, NS, CE, VEK, AE, CT, VTK, AT, Z, C, WK, EM, OML, OM, E$, P$, K$
1720 CLOSE
1730 GOTO 1820
1740 INPUT "combined electric, processing & kinematic data file name"; C$
1750 ON ERROR GOTO 1760: GOTO 1770
1760 RESUME 1740
1770 OPEN "I", #1, C$
1780 INPUT #1, EAD, DIP, COND, ELM, ETM, EVM, DT, IT
1790 INPUT #1, NS, CE, VEK, AE, CT, VTK, AT, Z, C, WK, EM, OML, OM, E$, P$, K$
1800 CLOSE
1810 EADR = EAD * CON: CR = C * CON: OMLR = OML * CON: OMR = OM * CON
1820 INPUT "required false alarm rate in false alarms per hour"; FAR
1830 PF = FAR * IT / 3600: REM false alarm probability
1840 Y = PF: IF PF > .5 THEN Y = 1 - Y: REM inverse normal approximation
1850 Y = SQR(LOG(1 / Y / Y))
1860 Y = Y - (I1 + Y * (I2 + I3 * Y)) / (1 + Y * (I4 + Y * (I5 + I6 * Y)))
1870 IF PF < .5 THEN Y = -Y
1880 ZP = -Y
1890 CHI = NS * (1 - 2 / 9 / NS + ZP * SQR(2 / 9 / NS)) ^ 3: REM inverse
chi-square approximation
```

```
1900 INPUT "electric noise in volts/meter"; SIG
1910 INPUT "maximum lateral range in meters"; LRM
1920 INPUT "lateral range step in meters"; ST
1930 IF ST <= LRM THEN GOTO 1950
1940 PRINT : PRINT "maximum step is " + STR$(LRM) + " meters": PRINT : GOTO
1920
1950 NC = 2 * INT(LRM / ST) + 1: REM number of lateral range function values
1960 IF NC <= 71 THEN GOTO 1980
1970 PRINT : PRINT "minimum step is " + STR$(LRM / 35) + " meters": PRINT :
GOTO 1920
1980 AIT = DT * NS: REM adjusted integration time in seconds
1990 W = WK * KON: REM electrometer relative speed in meters/second
2000 DS = W * DT: REM distance between samples on the relative track in meters
2010 X0 = -(NC - 1) / 2 * ST
2020 FOR I = 0 TO NC - 1
2030 X0(I) = X0
2040 X = X0: Y = Z
2050 GOSUB 3680
2060 R0 = R: R0(I) = R: REM target slant range at CPA in meters
2070 DELR = T: REM target depression angle complement at CPA in radians
2080 IF R0 = 0 THEN GOTO 2430: REM zero lateral range and vertical separation
2090 DMC = 4 * PI * COND
2100 DMF = EM / DMC / R0 ^ 3: REM dipole moment factor
2110 DIPR = DIP * CON: REM dipole depression angle in radians
2120 KR = CR - EADR: REM course referenced to the electrometer axis direction
2130 L = COS(OMR) * COS(KR - OMLR)
2140 M = COS(DELR) * COS(OMR) * SIN(KR - OMLR) - SIN(DELR) * SIN(OMR)
2150 N = -SIN(DELR) * COS(OMR) * SIN(KR - OMLR) - COS(DELR) * SIN(OMR)
2160 L1 = COS(DIPR) * COS(KR)
2170 M1 = COS(DELR) * COS(DIPR) * SIN(KR) - SIN(DELR) * SIN(DIPR)
2180 N1 = -SIN(DELR) * COS(DIPR) * SIN(KR) - COS(DELR) * SIN(DIPR)
2190 A2 = 2 * L * L1 - M * M1 - N * N1: REM anderson like function coefficient
2200 A1 = 3 * (N * L1 + L * N1): REM anderson like function coefficient
2210 A0 = 2 * N * N1 - L * L1 - M * M1: REM anderson like function coefficient
2220 SUM = 0: EMAX = 0: EMIN = 0
2230 FOR J = 0 TO NS - 1
2240 S = (J - (NS - 1) / 2) * DS: BA = S / R0: REM anderson like function
argument
2250 AF = 1 / (1 + BA * BA) ^ 2.5: REM anderson like function factor
2260 ESF = (A2 * BA * BA + A1 * BA + A0) * AF: REM electric signal factor
2270 ES(I, J) = DMF * ESF: REM electric signal value
2280 IF ES(I, J) > EMAX THEN EMAX = ES(I, J)
2290 IF ES(I, J) < EMIN THEN EMIN = ES(I, J)
2300 SUM = SUM + ESF * ESF
2310 NEXT J
2320 E(I) = EMAX - EMIN
2330 K(I) = SQR(SUM)
2340 VV = -ZP + DMF * SQR(SUM) / SIG
2350 LAM = DMF * DMF * SUM / (SIG * SIG): AA = NS + LAM: BB = 1 + LAM / (NS +
LAM)
2360 ZN = -SQR(2 * CHI / BB) + SQR(2 * AA / BB - 1): X1 = VV
2370 GOSUB 3740
```

```
2380 IF Y1 > 1 THEN Y1 = 1
2390 PDCC(I) = Y1: X1 = ZN
2400 GOSUB 3740
2410 IF Y1 > 1 THEN Y1 = 1
2420 PDSL(I) = Y1
2430 X0 = X0 + ST
2440 NEXT I
2450 C = C / 360: C = (C - INT(C)) * 360
2460 IF C < 0 THEN C = 360 + C
2470 OMLN = (OML + EAD) / 360: OMLN = (OMLN - INT(OMLN)) * 360
2480 IF OMLN < 0 THEN OMLN = 360 + OMLN
2490 PRINT : A$ = "a"
2500 INPUT "generate a program data file (y/n)"; A$
2510 IF A$ = "Y" OR A$ = "y" THEN GOTO 2530
2520 IF A$ = "N" OR A$ = "n" THEN GOTO 2860 ELSE GOTO 2500
2530 INPUT "program data file name"; D$
2540 ON ERROR GOTO 2550: GOTO 2560
2550 RESUME 2530
2560 OPEN "O", #1, D$
2570 WRITE #1, EAD, DIP, COND, ELM, ETM, EVM, DT, IT, AIT
2580 WRITE #1, NS, CE, VEK, AE, CT, VTK, AT, Z, C, WK, EM, OMLN, OM, FAR, PF,
SIG, ST
2590 WRITE #1, LRM, DS, NC, E$, P$, K$, C$
2600 FOR I = 0 TO NC - 1
2610 WRITE #1, X0(I), PDCC(I), PDSL(I), K(I), E(I), R0(I)
2620 NEXT I
2630 FOR I = 0 TO NC - 1
2640 FOR J = 0 TO NS - 1
2650 WRITE #1, ES(I, J)
2660 NEXT J
2670 NEXT I
2680 CLOSE
2690 GOTO 2860
2700 INPUT "program data file name"; D$
2710 ON ERROR GOTO 2720: GOTO 2730
2720 RESUME 2700
2730 OPEN "I", #1, D$
2740 INPUT #1, EAD, DIP, COND, ELM, ETM, EVM, DT, IT, AIT
2750 INPUT #1, NS, CE, VEK, AE, CT, VTK, AT, Z, C, WK, EM, OMLN, OM, FAR, PF,
SIG, ST
2760 INPUT #1, LRM, DS, NC, E$, P$, K$, C$
2770 FOR I = 0 TO NC - 1
2780 INPUT #1, X0(I), PDCC(I), PDSL(I), K(I), E(I), R0(I)
2790 NEXT I
2800 FOR I = 0 TO NC - 1
2810 FOR J = 0 TO NS - 1
2820 INPUT #1, ES(I, J)
2830 NEXT J
2840 NEXT I
2850 CLOSE
2860 PRINT : A$ = "a"
2870 INPUT "print encounter parameter values (y/n)"; A$
```

```
2880 IF A$ = "Y" OR A$ = "y" THEN GOTO 2900
2890 IF A$ = "N" OR A$ = "n" THEN GOTO 3290 ELSE GOTO 2870
2900 LPRINT
2910 LPRINT "program file name                                        " + N$
2920 LPRINT "program data file name                                   " + D$
2930 LPRINT "electric data file name                                  " + E$
2940 LPRINT "processing data file name                                " + P$
2950 LPRINT "kinematic data file name                                 " + K$
2960 LPRINT "combined electric, processing & kinematic data file name " + C$
2970 LPRINT "electrometer axis direction (decimal degrees)          "; SPC(2);
EAD
2980 LPRINT "electrometer axis depression angle (decimal degrees)   "; SPC(2);
DIP
2990 LPRINT "encounter medium conductivity (mhos/meter)             "; SPC(2);
COND
3000 LPRINT "electric longitudinal moment (ampere-meters)           "; SPC(2);
ELM
3010 LPRINT "electric transverse moment (ampere-meters)             "; SPC(2);
ETM
3020 LPRINT "electric vertical moment (ampere-meters)               "; SPC(2);
EVM
3030 LPRINT "sampling period (seconds)                              "; SPC(2);
DT
3040 LPRINT "integration time (seconds)                             "; SPC(2);
IT
3050 LPRINT "adjusted integration time (seconds)                    "; SPC(2);
AIT
3060 LPRINT "number of samples per encounter                        "; SPC(2);
NS
3070 LPRINT "electrometer course (decimal degrees)                  "; SPC(2);
CE
3080 LPRINT "electrometer speed (knots)                             "; SPC(2);
VEK
3090 LPRINT "electrometer altitude (meters)                         "; SPC(2);
AE
3100 LPRINT "target course (decimal degrees)                        "; SPC(2);
CT
3110 LPRINT "target speed (knots)                                   "; SPC(2);
VTK
3120 LPRINT "target depth (meters)                                  "; SPC(2);
AT
3130 LPRINT "electrometer relative course (decimal degrees)         "; SPC(2);
C
3140 LPRINT "electrometer relative speed (knots)                    "; SPC(2);
WK
3150 LPRINT "electrometer-target vertical separation (meters)       "; SPC(2);
Z
3160 LPRINT "electric dipole moment (ampere-meters)                 "; SPC(2);
EM
3170 LPRINT "dipole moment azimuth (decimal degrees)                "; SPC(2);
OMIN
```

```
3180 LPRINT "dipole moment depression angle (decimal degrees)       "; SPC(2);
OM
3190 LPRINT "distance between samples on the relative track (meters)"; SPC(2);
DS
3200 LPRINT "false alarm rate (false alarms per hour)               "; SPC(2);
FAR
3210 LPRINT "false alarm probability                                "; SPC(2);
PF
3220 LPRINT "electric noise (volts/meter)                           "; SPC(2);
SIG
3230 LPRINT "maximum lateral range (meters)                         "; SPC(2);
LRM
3240 LPRINT "lateral range step (meters)                            "; SPC(2);
ST
3250 LPRINT "number of lateral range function values                "; SPC(2);
NC
3260 FOR I = 0 TO 30
3270 LPRINT
3280 NEXT I
3290 PRINT : A$ = "a"
3300 INPUT "print lateral range function values (y/n)"; A$
3310 IF A$ = "Y" OR A$ = "y" THEN GOTO 3330
3320 IF A$ = "N" OR A$ = "n" THEN GOTO 3410 ELSE GOTO 3300
3330 LPRINT D$; " lateral range function values"
3340 LPRINT : LPRINT
3350 LPRINT "L          p(cc)        p(sl)         E             R
    k"
3360 LPRINT "meters                                volts/meter   meters"
3370 LPRINT
3380 FOR I = 0 TO NC - 1
3390 LPRINT X0(I); TAB(10); PDCC(I); TAB(24); PDSL(I); TAB(38); E(I); TAB(52);
R0(I); TAB(70); K(I)
3400 NEXT I
3410 PRINT : A$ = "a"
3420 INPUT "print electric signal values (y/n)"; A$
3430 IF A$ = "Y" OR A$ = "y" THEN GOTO 3450
3440 IF A$ = "N" OR A$ = "n" THEN GOTO 3630 ELSE GOTO 3420
3450 PRINT : PRINT "identify signal by encounter lateral range index"
3460 PRINT "lateral range equals the index times " + STR$(ST) + " meters"
3470 PRINT "index values: -" + STR$((NC - 1) / 2) + " to " + STR$((NC - 1) /
2)
3480 INPUT "lateral range index"; K: I = K + (NC - 1) / 2
3490 LPRINT : LPRINT : LPRINT
3500 LPRINT D$; " electric signal values"
3510 LPRINT "for a lateral range of "; K * ST; " meters"
3520 LPRINT
3530 LPRINT "relative CPA distance"; TAB(35); "electric signal"
3540 LPRINT "s in meters"; TAB(35); "Es in volts/meter"
3550 LPRINT
3560 FOR J = 0 TO NS - 1
3570 LPRINT (J - (NS - 1) / 2) * DS; TAB(35); ES(I, J)
3580 NEXT J
```

```
3590 PRINT : A$ = "a"
3600 INPUT "print electric signal values for a different lateral range (y/n)";
A$
3610 IF A$ = "Y" OR A$ = "y" THEN GOTO 3450
3620 IF A$ = "N" OR A$ = "n" THEN GOTO 3630 ELSE GOTO 3600
3630 PRINT : A$ = "a"
3640 INPUT "continue to use the program (y/n)"; A$
3650 IF A$ = "Y" OR A$ = "y" THEN GOTO 10
3660 IF A$ = "N" OR A$ = "n" THEN GOTO 3670 ELSE GOTO 3640
3670 END
3680 R = SQR(X * X + Y * Y): REM rectangular to polar conversion
3690 IF R = 0 THEN T = 0: RETURN
3700 IF ABS(X / R) = 1 THEN Q = SGN(X) * (PI / 2) ELSE Q = ATN(X / R / SQR(1 -
X * X / R / R))
3710 IF ABS(Y / R) = 1 THEN T = (PI / 2) * (1 - SGN(Y)) ELSE T = (PI / 2) -
ATN(Y / R / SQR(1 - Y * Y / R / R))
3720 IF Q < 0 THEN T = 2 * PI - T
3730 RETURN
3740 Y1 = X1: IF X1 < 0 THEN Y1 = -Y1: REM normal approximation
3750 G = 1 / (1 + Q0 * Y1)
3760 Y1 = EXP(-Y1 * Y1 / 2) / SQR(2 * PI) * G * (Q1 + G * (Q2 + G * (Q3 + G *
(Q4 + G * Q5))))
3770 IF X1 >= 0 THEN Y1 = 1 - Y1
3780 RETURN
```

References

1. "Mad Signal Processing Study," C2-61009-2, Texas Instruments, Inc., Dallas, Texas, April 1961.

2. Handbook of Mathematical Functions, National Bureau of Standards, Applied Mathematics Series 55, Washington, D.C., June 1964.

3. "Magnetic Airborne Detector Program," Summary Technical Report of Division 6, National Defense Research Committee, Vol. 5, Washington, D.C., 1946.

4. Anderson, J. E., "Magnetic Airborne Detection Frequency Responses," NADC-EL-47-50, Naval Air Development Center, Johnsville, PA, October 1949.

5. Schluckebier, D. C., "A Comparison of Three Magnetic Anomaly Detection (MAD) Models," Thesis, Naval Postgraduate School, March 1984.

6. Forrest, R. N., "A Program to Compute Magnetic Anomaly Detection Probabilities,"NPS71-88-001 Revised), Naval Postgraduate School, Monterey, California 93943, May 1988 (Revised October 1988).

Initial Distribution List

	Copies
Director of Research Administration (Code 012) Naval Postgraduate School Monterey, CA 93943-5000	1
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Library (Code 0142) Naval Postgraduate School Monterey, CA 93943-5000	2
Commander Submarine Development Squadron 12 Naval Submarine Base, New London Groton, CT 06349	1
Commander Surface Warfare Development Group Naval Amphibious Base, Little Creek Norfolk, VA 23521	1
Naval Air Development Center Johnsville, PA 18974	1
Naval Surface Weapons Center White Oak Silver Spring, MD 20910	1
Naval Underwater Systems Center Newport, RI 02840	1
Naval Underwater Systems Center New London, CT 06320	1
Naval Technical Intelligence Center 4301 Suitland Road Washington, D. C. 20390	1
Naval Research Laboratory Washington, D. C. 20375	1
Center for Naval Analysis 4401 Ford Avenue P. O. Box 16268 Alexandria, VA 22302-0268	1

Naval Ocean Systems Center 1
San Diego, CA 92132

Naval Postgraduate School
Monterey, CA 93943-5000
Code 55Fo 30